Dielectric functions for the Vlasov-Landau equation

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Longitudinal and transverse dielectric functions are derived from the Vlasov equation with the Landau ion-electron collision operator. The linear collisional propagator has been computed with the use of continuous fractions [Phys. Fluids **30**, 1353 (1987)]. The model gives results which are valid in all the frequency and collisionality range. For Maxwellian plasmas, the dispersion relations of Langmuir and electromagnetic waves are explicitly computed with thermal corrections up to the fourth order. For the transverse waves, the results obtained are in good agreement with those deduced from the collisional Dawson-Oberman model [Phys. Fluids **5**, 517 (1962)]. A numerical solution of the longitudinal damping rate is obtained. The dispersion relations for weakly anisotropic plasmas are studied. It is shown in particular that, for the collisional damping rates, the corrections due to the anisotropic effects are of the same order as the thermal corrections.

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I. INTRODUCTION

The derivation of the dispersion relations for electromagnetic and Langmuir waves throughout the collisionality range has a great importance in laser-plasma interaction and astrophysics. These dispersion relations express the frequency ω_r and the damping rate γ as function of the wave number k. These parameters are very important data to describe many physical phenomena (electromagnetic wave absorption, parametric instabilities, etc.). Many works in this field have been reported in the literature. They can be classified into collisional and collisionless asymptotic approaches. In the collisionless limit, by solving the Vlasov-Poisson equations, Landau [1] established the well-known Landau damping due to the wave-particle resonant interaction. For Maxwellian plasmas, the high-frequency collisionless dielectric function is given by

$$\epsilon_{ij}(\mathbf{k},\omega)=\left[\delta_{ij}-\frac{k_i k_j}{k^2}\right]\epsilon_T(k,\omega)+\frac{k_i k_j}{k^2}\epsilon_L(k,\omega) ,$$

where the longitudinal and the transverse parts read

$$\epsilon_L=1-\frac{\omega_p^2}{\omega^2}\left[1+3(k\lambda_D)^2+\frac{i}{\sqrt{2\pi}(k\lambda_D)^3}\exp\left[-\frac{\omega_r^2}{2k^2v_t^2}\right]\right] , \quad (1)$$

$$\epsilon_T=1-\frac{\omega_p^2}{\omega^2} , \quad (2)$$

where $\lambda_D=(\epsilon_0 T/ne^2)^{1/2}$ is the Debye length, $v_t=(T/m)^{1/2}$ the thermal velocity, $\omega_p=(ne^2/m\epsilon_0)^{1/2}$ the electronic plasma frequency, and $\omega=\omega_r+i\gamma$ the complex frequency of plasma modes. In the collisional limit many results have been derived for the damping rate using quantum mechanics [2–4] or classical kinetics [5–10] formalisms. In each of these studies, the main purpose was to derive the collisional damping rate. For the electromagnetic waves, the most important result was obtained by Dawson and Oberman [7]. This model treats exactly the momentum exchange between the ion and the electron components and therefore is valid to describe high-frequency phenomena [Sec. II B]. Using kinetic theories Shkarofsky [8], McBride [9], and more recently, Jasperse and Basu [10] have shown that the total damping rate of the electron plasma waves is approximately the sum of the Landau and the collisional damping rates:

$$\gamma_{\text{total}}=-\left[\frac{\pi}{8}\right]^{1/2}\omega_p(k\lambda_D)^3\exp\left[-\frac{\omega_r^2}{2(k\lambda_D)^2}\right]-\frac{v_t}{3\sqrt{2\pi}\lambda_0} , \quad (3)$$

where

$$\lambda_0=\frac{4\pi\epsilon_0^2T^2}{ne^4\ln\Lambda Z} \quad (4)$$

is the electronic mean free path. In this paper our purpose is to derive exact semicollisional longitudinal $\epsilon_L(\mathbf{k},\omega)$ and transverse $\epsilon_T(\mathbf{k},\omega)$ dielectric functions in anisotropic plasmas with the use of the Vlasov-Landau kinetic equation. In our calculation we use the local approximation that the spatial inhomogeneity effects are negligible. Furthermore, we do not take into account the electron-electron correlations (high Z limit) and hydrodynamic ion motion is neglected. The methods for the semicollisional propagator calculation used in this work are those derived in Ref. [11] which were previously [12] employed for the Weibel instabilities analysis in lasercreated plasmas. To our knowledge, no exact work on semicollisional dispersion relations, i.e., valid in an intermediate regime, where both collisional and collisionless damping are simultaneously present, has been reported in the literature up to now, but only approximate approaches. Our results describe the continuous transition between the collisional and the collisionless limit with respect to the dimensionless parameter $k\lambda_D$. This paper is organized as follows.

First, we describe in Sec. II the normalized Vlasov-Landau equation and discuss the approximations used in our model. Then we analyze the high-frequency validity of the Vlasov-Landau equation with a brief description of the Dawson-Oberman model [7]. Section III is devoted to the derivation of the semicollisional propagators. In Sec. IV we compute the longitudinal and transverse dielectric functions. The dispersion relations of Langmuir and electromagnetic waves are presented in an explicit form for Maxwellian plasma in Sec. V. A comparison with the Dawson-Oberman model is also performed. In Sec. VI the contribution of anisotropic effects is computed for both modes by using the Chapman-Enskog expansion to compute the secular distribution function. A summary of the whole work will be, finally, presented and will be followed by a discussion about some specific future extensions of the present results.

II. THE MODEL AND ITS CONDITIONS OF VALIDITY

In this section we deal with the normalized Vlasov-Landau equation and some approximations used in our model. We also discuss the high-frequency validity of the Vlasov-Landau model by comparing it with the Dawson-Oberman model (referred to hereafter respectively as VL and DO models).

A. Vlasov-Landau equation and approximations

The VL Equation for the electrons in the ion frame reads [13]

$$\frac{\partial f_e}{\partial t}+\mathbf{v}\cdot\frac{\partial f_e}{\partial \mathbf{r}}-\frac{e}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\frac{\partial f_e}{\partial \mathbf{v}}=\sum_{n=e,i}\frac{e^4(\ln\Lambda)Z^2}{8\pi\epsilon_0^2 m}\frac{\partial}{\partial v_\beta}\int d\mathbf{v}'\left[\frac{\partial f_e(\mathbf{v})}{\partial v_\gamma}\frac{f_n(\mathbf{v}')}{m}-\frac{f_e(\mathbf{v})}{M_i}\frac{\partial f_n(\mathbf{v}')}{\partial v'_\gamma}\right]U_{\beta\gamma} , \tag{5}$$

where the right-hand side is the Landau collision operator defined with the following parameters:

$$U_{\beta\gamma}=\frac{1}{u^3}(u^2\delta_{\beta\gamma}-u_\beta u_\gamma), \text{ with } u_\beta=v_\beta-v'_\beta .$$

The other quantities have their usual meaning. Let us perform some approximations used in our model. In the left-hand side of Eq. (5) the ion fluid velocity is neglected ($v_{\text{ionic}} \ll v_t$). In the right-hand side we expand the tensor $U_{\beta\gamma}$ with respect to the ion velocity and keep the leading terms, i.e., $U_{\beta\gamma}=v^{-3}(v^2\delta_{\beta\gamma}-v_\beta v_\gamma)$. We neglect the electron-electron correlations (high Z limit) and the energy exchange between electrons and ions. This gives us the following simple Landau collision operator (pitch angle scattering operator) which describes the momentum exchange between the ions and the electrons:

$$C_{ei}(f)=\frac{v_t^4}{2\lambda_0 v^3}\frac{\partial}{\partial v_i}(v^2\delta_{ij}-v_i v_j)\frac{\partial f}{\partial v_j} \tag{6}$$

(with the subscript in f dropped). It should be pointed out that this operator does not explicitly conserve the momentum, the ions being assumed fixed.

Let us now derive the linearized VL equation. For this we set

$$f=f_s+\delta f\exp(-i\omega t+i\mathbf{k}\cdot\mathbf{r})$$

and

$$(\mathbf{E},\mathbf{B})=(\mathbf{E}_s,\mathbf{B}_s)+(\delta\mathbf{E},\delta\mathbf{B})\exp(-i\omega t+i\mathbf{k}\cdot\mathbf{r}) ,$$

where $\mathbf{k}$ is a real wave vector and ω a complex frequency. δf, $\delta\mathbf{E}$, and $\delta\mathbf{B}$ are the perturbed quantities and f_s, $\mathbf{E}_s$, and $\mathbf{B}_s$ the unperturbed ones. Furthermore, we assume that the local approximation, $kL \gg 1$, is fulfilled (L is a characteristic plasma scale length). With the use of these approximations we obtain the following VL equation written in the frame $(\mathbf{u},\mathbf{w},\mathbf{z})$ such as $\mathbf{k}=k\mathbf{w}$ (see Fig. 1):

$$(\Omega+\mu q-C_\perp)\delta f(y,\Omega,\mathbf{q})=\delta S(\delta\xi,\delta\boldsymbol{\beta},f_s) , \tag{7}$$

where the ion-electron collision operator and the source term are given, respectively, by

$$C_\perp(\mu,\phi)=\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu}+\frac{1}{1-\mu^2}\frac{\partial^2}{\partial\phi^2} , \tag{8}$$

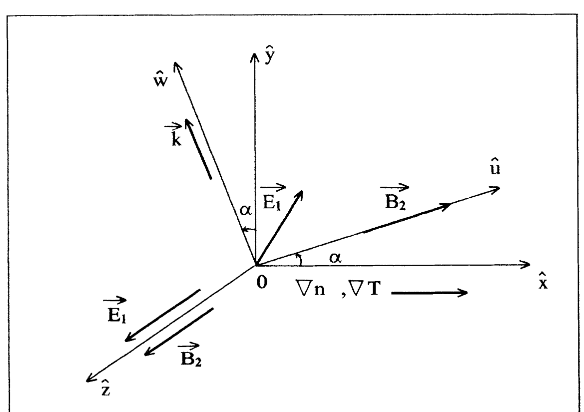

FIG. 1. Geometry of the first and the second mode (respectively $\mathbf{k},\mathbf{E}_1,\mathbf{B}_1$ and $\mathbf{k},\mathbf{E}_2,\mathbf{B}_2$) in the $(\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}})$ frame. The $(\hat{\mathbf{u}},\hat{\mathbf{w}},\hat{\mathbf{z}})$ frame is obtained from the $(\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}})$ one by an α rotation with respect to the $\hat{\mathbf{z}}$ axis. The density and the temperature inhomogeneities are along the $\hat{\mathbf{x}}$ axis.

and

$$\delta S(\delta\xi,\delta\beta,f_s)=4\sqrt{6}y^2\frac{\partial f_s}{\partial y}\left[\delta\xi+2y^{1/2}\frac{\mathbf{v}}{v}\times\delta\beta\right]\cdot\frac{\mathbf{v}}{v}\ .$$

The variables defined as dimensionless quantities are

$y=mv^2/2T,\quad \Omega=-i\omega 4\sqrt{2}\lambda_0 y^{3/2}/v_t,\quad \mathbf{q}=i8\lambda_0 y^2\mathbf{k}\ ,$

$\delta\xi=\sqrt{2/3}(e\lambda_0/T)\delta\mathbf{E},\quad$ and $\quad\delta\beta=\sqrt{2/3}(e\lambda_0/mv_t)\delta\mathbf{B}\ .$

The spherical angular variables are

$$\mu=\frac{v_w}{v}\quad\text{and}\quad\cos^2\phi=\frac{v_z^2}{v^2(1-\mu^2)}\ .$$

We can note that the source term S involves the perturbed electromagnetic fields $\delta\xi$ and $\delta\beta$ which are related by the Maxwell equations' to the perturbed distribution function.

B. High-frequency validity of the model

It is well known that the Landau collision operator is derived from the first two Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) equations which involve the reduced one-particle and two-particle distribution functions. The other basic assumption (Bogoliubov hypothesis) is that the two-point correlation function (which describes the collisions) relaxes on a time scale very short as compared to the time scale on which the reduced one-particle distribution function f relaxes. Therefore, the VL model should not be rigorously valid in the high-frequency limit ($\omega_r\geq\omega_p$). In order to describe high-frequency phenomena some authors [5–7] have used more accurate models where the Bogoliubov hypothesis is not used. One of these models, DO [7], which describes exactly the ion-electron collisions and uses the same approximations (electron-electron correlations neglected and ions at rest) as the present model, will be used here for comparison. The DO model considers that the electrons are governed by the Vlasov equation with an electric field, that is, the sum of an oscillating part and an electrostatic part generated by the discrete ion distribution. The ion-ion correlations are assumed to be externally given. In Ref. [14], it is shown that for low frequency ($\omega_r\ll\omega_p$) the DO collision operator takes the Landau form if one assumes the ion-spectrum turbulence isotropic and the electronic mean free path [Eq. (4)] to read

$$\lambda_0=4\pi\left[\int k^3\left|\frac{e\phi_s(k)}{T}\right|^2 dk\right]^{-1}\ , \tag{9}$$

where $\phi_s(k)=S_s(k)/\epsilon_T(k,0)$ is the ion microscopic turbulence defined by

$$|S_s(k)|^2=\frac{Z^2e^4}{k^4\epsilon_0^2}\left[n_i+\sum_{\substack{i,j\\ i\neq j}}\exp[i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_j)]\right]\ .$$

If we assume a thermal ion correlation level, i.e.,

$$|S_s(k)|^2=\frac{Z^2e^4}{k^4\epsilon_0^2}n_i\frac{1+(k\lambda_D)^2}{Z+1+(k\lambda_D)^2}\ ,$$

Eq. (9) gives Eq. (4). In the high-frequency limit ($\omega_r\geq\omega_p$), applied to the computation of ponderomotive effects [14], the two models differ qualitatively by a factor called the generalized Gaunt factor [Eq. (61) in Ref. [14]] which takes into account the high-frequency effects. This factor is approximately one for $\omega_r\simeq\omega_p$ and for large Coulomb logarithms. This value is in general equal to one, however, it no longer takes this value in the cases where the ion acoustic turbulence is far from the thermal level and for ω_r slightly exceeding ω_p in a strongly dominated collision regime ($\mathbf{v}\cdot\nabla\ll v_t/\lambda_0$). In Sec. V B we corroborate the good agreement between those two models in the high-frequency range by comparing the damping rates.

III. VLASOV-LANDAU COLLISIONAL PROPAGATOR

The reduced VL equation [Eq. (7)] admits the formal solution

$$\delta f(\mu,\phi,y)=G(\mu,\phi)\delta S(\delta\xi,\delta\beta,f_s)\ ,$$

where $G(\mu,\phi)=(\Omega+\mu q-C_\perp)^{-1}$ is the two-dimensional (2D) VL propagator. By using continuous fractions, used before for the computation of the plasma dispersion functions [15], we can explicitly compute the propagator $G(\mu,\phi)$ on the basis of spherical harmonics $Y_n^{(m)}(\mu,\phi)$ that are the eigenfunctions of the Landau collision operator $C_\perp$ [Eq. (8)]. Let us briefly outline the computation method for this propagator. The expansion of the perturbed quantities is made on the $Y_n^{(m)}(\mu,\phi)$ basis:

$$(\delta f(\mu,\phi,y),\delta S(\mu,\phi,y))=\sum_{n=0}^{\infty}\sum_{m=-n}^{+n}[\delta f_n^{(m)}(y),\delta S_n^{(m)}(y)]\times Y_n^{(m)}(\mu,\phi)\ ,$$

whereas the unperturbed distribution function is expanded on the Legendre polynomial basis $P_n(v_x/v)$:

$$f_s\left[y,\frac{v_x}{v}\right]=\sum_{n=0}^{\infty}\sqrt{2n+1}P_n\left[\frac{v_x}{v}\right]f_s^{(n)}(y)\ . \tag{10}$$

By projecting Eq. (7) on the associated Legendre polynomial sub-basis corresponding to a given m, we obtain formally $\delta f^{(m)}=G^{|m|}(\mu)\delta S^{(m)}$. The components $\delta f^{(m)}$ and $\delta S^{(m)}$ are two infinite column vectors and the subpropagator $G^{|m|}(\mu)$ a symmetric infinite square matrix. Using the standard recurrence relation [16] of the continuous fractions, the (n,m) distribution function component can be expressed after some lengthy calculations as

$$\delta f_n^{(m)}(y)=\sum_{i=0}^{\infty}G_{ni}^{|m|}\delta S_i^{(m)}\ , \tag{11}$$

where

$$G_{ni}^{|m|}(n>i)=(-q)^{n-i}V_{i+1}^{|m|}\cdots V_n^{|m|}F_{i+1}^{|m|}\cdots F_n^{|m|}G_{ii}^{|m|}$$

and

$$G_{ii}^{|m|}=D_i^{|m|}F_0^{|m|}\cdots F_i^{|m|}\ .$$

The $D_i^{|m|}$ quantities are defined by

$$D_i^{|m|}=[\Omega+(i+|m|)(i+|m|-1)]D_{i-1}^{|m|}-q^2(V_{i-1}^{|m|})^2D_{i-2}^{|m|}\ ,$$

with

$$D_{-1}^{|m|}=0,\quad D_0^{|m|}=1\ ,$$

and

$$V_i^{|m|}=\{i(i+2|m|)/[4(i+|m|)^2-1]\}^{1/2}\ ,\qquad i=0,1\ldots.$$

The continuous fractions are defined by the following recursive relation:

$$F_i^{|m|}=[\Omega+(i+|m|)(i+1+|m|)-q^2(V_{i+1}^{|m|})^2F_{i+1}^{|m|}]^{-1}\ . \tag{12}$$

In the next section we explicitly derive the $\delta f_0^{(0)}$ and $\delta f_1^{(1)}$ components, which are useful for the computation of the Maxwell equations' source terms (i.e., the charge and the current densities).

IV. DIELECTRIC FUNCTIONS

Let us now derive the longitudinal and transverse VL dielectric functions valid in the whole k range. In the standard notations the dielectric functions are written

$$\epsilon_{ij}(\mathbf{k},\omega)=\delta_{ij}-i\frac{\omega_p^2}{\omega}\int_{-\infty}^{+\infty}G\frac{v_iv_j}{v}\frac{\partial f_s}{\partial v}d\mathbf{v}\ , \tag{13}$$

with the normalization condition $\int_{-\infty}^{+\infty}f_s d\mathbf{v}=1$. G is the propagator which takes, in the collisionless limit, the well-known form

$$G=(-i\omega+i\mathbf{k}\cdot\mathbf{v})^{-1}\ .$$

We assume that the longitudinal and transverse modes propagate in an inhomogeneous plasma $(n(x),T(x))$, with a wave vector $\mathbf{k}=k\mathbf{w}$ (Fig. 1). For the electrostatic wave $\delta\mathbf{E}=\delta E\mathbf{w}$, the normalized Poisson equation is

$$\delta E=-\frac{16\sqrt{4\pi}v_te^2\lambda_0^2y^2}{\sqrt{3}m\epsilon_0q}\int_0^\infty y^{1/2}\delta f_0^{(0)}dy\ .$$

We deduce from Eq. (13) the longitudinal dielectric function

$$\epsilon_L=1+\frac{16\sqrt{4\pi}v_te^2\lambda_0^2y^2}{\sqrt{3}m\epsilon_0q}\int_0^\infty y^{1/2}\left[\frac{\delta f_0^{(0)}}{\delta E}\right]dy\ . \tag{14}$$

We now consider the electromagnetic wave. Taking the wave vector $\mathbf{k}$ in the x-y plane for symmetry considerations, we define one mode with the magnetic field along the z axis and the other one with the magnetic field in the x-y plane (see Fig. 1). Let us call them mode 1 and 2, respectively. For mode 1 the Faraday and Ampère laws are

$$2i\mu_0\sqrt{8\pi/3}v_t^4e\int_0^\infty y\delta f_1^{(1)}dy-i\sqrt{3/2}\left[\frac{\omega T}{ec^2\lambda_0}\right]\delta E_u=\sqrt{3/2}\left[\frac{\sqrt{mT}q}{8e\lambda_0^2y^2}\right]\delta B$$

and

$$\delta E_u=-(\omega/kv_t)\delta B\ .$$

For mode 2

$$\mu_0\sqrt{8\pi/3}v_t^4e\int_0^\infty y(\delta f_1^{(1)}-\delta f_1^{(-1)})dy-i\sqrt{3/2}\left[\frac{\omega T}{ec^2\lambda_0}\right]\delta E_z=-\sqrt{3/2}\left[\frac{\sqrt{mT}q}{8e\lambda_0^2y^2}\right]\delta B$$

and

$$\delta E_z=(\omega/kv_t)\delta B\ .$$

Thus from Eq. (13) we obtain the following transverse dielectric functions:

$$\epsilon_{T_1}=1-\frac{8\sqrt{\pi}v_t^2e^2\lambda_0}{3\omega\epsilon_0m}\int_0^\infty y\left[\frac{\delta f_1^{(1)}}{\delta E_u}\right]dy\ , \tag{15}$$

$$\epsilon_{T_2}=1+\frac{i4\sqrt{\pi}v_t^2e^2\lambda_0}{3\omega\epsilon_0m}\int_0^\infty y\left[\frac{\delta f_1^{(1)}-\delta f_1^{(-1)}}{\delta E_z}\right]dy\ , \tag{16}$$

where $\delta f_0^{(0)}$ and $\delta f_1^{(\pm1)}$ are functions defined by Eq. (11). Equations (14)–(16) are the semicollisional VL dielectric functions. We note that the truncation of $\delta f_0^{(0)}$ and $\delta f_1^{(\pm1)}$ at any given order is equivalent to the truncation of the unperturbed anisotropic distribution function f_s. As regards the continuous fractions, the truncation is equivalent to the expansion of the dielectric function with respect to the parameter (kv_t/ω_r).

V. MAXWELLIAN DISPERSION RELATIONS

In this section we deal with the semicollisional dispersion relation of the Langmuir and electromagnetic waves in a Maxwellian plasma, i.e., $f_s=f_M=(m/2\pi T)^{3/2}\exp(-y)$. Therefore we neglect for the moment anisotropic effects, which will be considered in the next section. The explicit derivation of the perturbed components' distribution functions for both modes gives

$$\delta f_0^{(0)}=4\sqrt{8\pi/3}qy^2F_0^{(0)}F_1^{(0)}(\Omega,q)\times(m/2\pi T)^{3/2}\exp(-y)\delta E_w$$

and

$$\delta f_1^{(1)}=-i8\sqrt{\pi}y^2F_0^{(1)}(\Omega,q)(m/2\pi T)^{3/2}\exp(-y)\delta E_u\ ,$$

where $F_0^{(0)}$, $F_0^{(1)}$, and $F_1^{(0)}$ are continuous fractions defined by Eq. (12). The derivation of isotropic dielectric functions is straightforward from Eqs. (14)–(16):

$$\epsilon_L = 1 + \frac{128\lambda_0^2\omega_p^2}{3\sqrt{\pi}v_t^2}\int_0^\infty y^{9/2}F_0^{(0)}F_1^{(0)}(\Omega,q) \times \exp(-y)dy \ , \quad (17)$$

$$\epsilon_T = 1 + \frac{i32\lambda_0\omega_p^2}{3\sqrt{2\pi}v_t\omega}\int_0^\infty y^3F_0^{(1)}(\Omega,q)\exp(-y)dy \ . \quad (18)$$

We note that for isotropic plasmas, $\epsilon_{T_1} = \epsilon_{T_2}$. We have checked that for spatially uniform modes the relation $\epsilon_L(0,\omega) = \epsilon_T(0,\omega)$ is fulfilled. Now from Eqs. (17) and (18) we can deduce the dispersion relation for the Langmuir and electromagnetic waves, which are valid in the whole collisionality regime, by using the equations

$$\epsilon_L(\mathbf{k},\omega) = 0 \quad (19)$$

and

$$\epsilon_T(\mathbf{k},\omega) = (kc/\omega)^2 \ . \quad (20)$$

A. Langmuir dispersion relation

First, let us consider the dispersion relation in the collisional and the high-frequency ($kv_t/\omega_r \ll 1$) approximations. The expansion of the continuous fractions' product $F_0^{(0)}F_1^{(0)}$ up to the fourth order with respect to parameter kv_t/ω_r gives

$$F_0^{(0)}F_1^{(0)} = -\Omega_r^{-2}\left[1 + \frac{3}{5}\left[\frac{q}{\Omega_r}\right]^2 + \frac{3}{7}\left[\frac{q}{\Omega_r}\right]^4\right] + 2i\Omega_r^{-3}\left[1 + \Gamma + 2\left[\frac{q}{\Omega_r}\right]^2 + \frac{6}{5}\Gamma\left[\frac{q}{\Omega_r}\right]^2 + 3\left[\frac{q}{\Omega_r}\right]^4 + \frac{9}{7}\Gamma\left[\frac{q}{\Omega_r}\right]^4\right] .$$

The notation used is $\Omega = \Omega_r + i\Gamma$. Keeping the real part of Eq. (19) we obtain the usual high-frequency Langmuir dispersion relation

$$\omega_r = \omega_p[1 + 3(k\lambda_D)^2 + 6(k\lambda_D)^4 + \cdots]^{1/2} \ . \quad (21)$$

The imaginary part of Eq. (19) gives the collisional damping rate

$$\gamma_L = -\frac{v_t}{3\sqrt{2\pi}\lambda_0}[1 - 2(k\lambda_D)^2 - 3(k\lambda_D)^4 + \cdots] \ . \quad (22)$$

We recover exactly the result deduced in Ref. [10], where the propagator expansion method has been used and applied to the Vlasov-Balescu-Lenard kinetic equation. This similarity is due to the fact that both Landau and Balescu-Lenard operators are of the Fokker-Planck type; therefore, when applied to Maxwellian plasmas, their linearized forms are equivalent. In the low-frequency limit ($kv_t/\omega_r \gg 1$) the results are more straightforward: The real part of Eq. (19) describes the static shielding of the ionic potential by electrons:

$$\mathrm{Re}(\epsilon_L) = 1 + (k\lambda_D)^{-2} \ .$$

The imaginary part gives, in the collisional approximation the quasistatic conductivity, i.e., $\sigma = \epsilon_0\omega_r \mathrm{Im}(\epsilon_L)$. In this approximation $\mathrm{Im}(F_0^{(0)}F_1^{(0)}) = 1/2\Omega_r$. Hence, substituting this result in Eq. (17), we find

$$\sigma = \frac{64\sqrt{2\pi}\epsilon_0^2T^{3/2}}{e^2\sqrt{m}\ \ln(\Lambda)Z} \ . \quad (23)$$

Thus we recover the high Z Spitzer-Härm [17] conductivity. The damping rate deduced from Eq. (19), which is valid for all the collisionality regime, is

$$\gamma_L = \frac{(\omega_p\lambda_0/\lambda_D)\int_0^\infty y^2\exp(-y)\mathrm{Re}(F_0^{(0,\nu)})dy}{\int_0^\infty y^2\exp(-y)\mathrm{Im}(F_0^{(0)})dy} \times \left[1 - \frac{(\lambda_0/\lambda_D)\int_0^\infty y^2\exp(-y)\mathrm{Re}(F_0^{(0,\gamma)})dy}{\int_0^\infty y^2\exp(-y)\mathrm{Im}(F_0^{(0)})dy}\right]^{-1} , \quad (24)$$

where the continuous fractions used are defined as

$$F_l^{(0)} = \Gamma\ \mathrm{Re}(F_l^{(0,\gamma)}) + \mathrm{Re}(F_l^{(0,\nu)}) + i\ \mathrm{Im}(F_l^{(0)}) \ ,$$

with

$$\mathrm{Im}(F_l^{(0)}) = \left[4\sqrt{2}\left[\frac{\lambda_0\omega_r}{\omega_p\lambda_D}\right]y^{3/2} - 64(k\lambda_0)^2y^4 \times \frac{(l+1)^2}{4(l+1)^2-1}\mathrm{Im}(F_{l+1}^{(0)})\right]^{-1} ,$$

$$\mathrm{Re}(F_l^{(0,\gamma)}) = [\mathrm{Im}(F_l^{(0)})]^2\left[4\sqrt{2}y^{3/2}\frac{\lambda_0}{\lambda_D} + \frac{64(l+1)^2y^4}{4(l+1)^2-1}(k\lambda_0)^2 \times \mathrm{Re}(F_{l+1}^{(0,\gamma)})\right] ,$$

and

$$\mathrm{Re}(F_l^{(0,\nu)}) = [\mathrm{Im}(F_l^{(0)})]^2\left[l(l+1) + \frac{64(l+1)^2y^4}{4(l+1)^2-1}(k\lambda_0)^2 \times \mathrm{Re}(F_{l+1}^{(0,\nu)})\right] .$$

In the intermediate regime we have numerically computed (Fig. 2) the damping rate [Eq. (24)] with respect to the parameter $k\lambda_D$ for different collisionality parameters (λ_0/λ_D). We emphasize the continuous transition between the asymptotic collisional limit and the collisionless one characterized by the resonant wave-particle energy transfer. In the usual approaches the resonant Landau effect can be studied in isolation and appears naturally as a pole in the damping rate γ_L. Here the Landau and the collisional damping are intimately mixed and it is impossible to isolate the respective contribution. However, the Landau damping is revealed by a peak of $\mathrm{Re}(F_0^{(0,\nu)})$ at the resonant velocity $y_r = (\omega_r / k v_t)^2/2$. We have checked that this peak is strongly enhanced when the phase velocity decreases due to the correlated increase of the resonant electronic population. Of course the relative importance of the resonant Landau effect increases with λ_0 (low collisional regime) as displayed in Fig. 2.

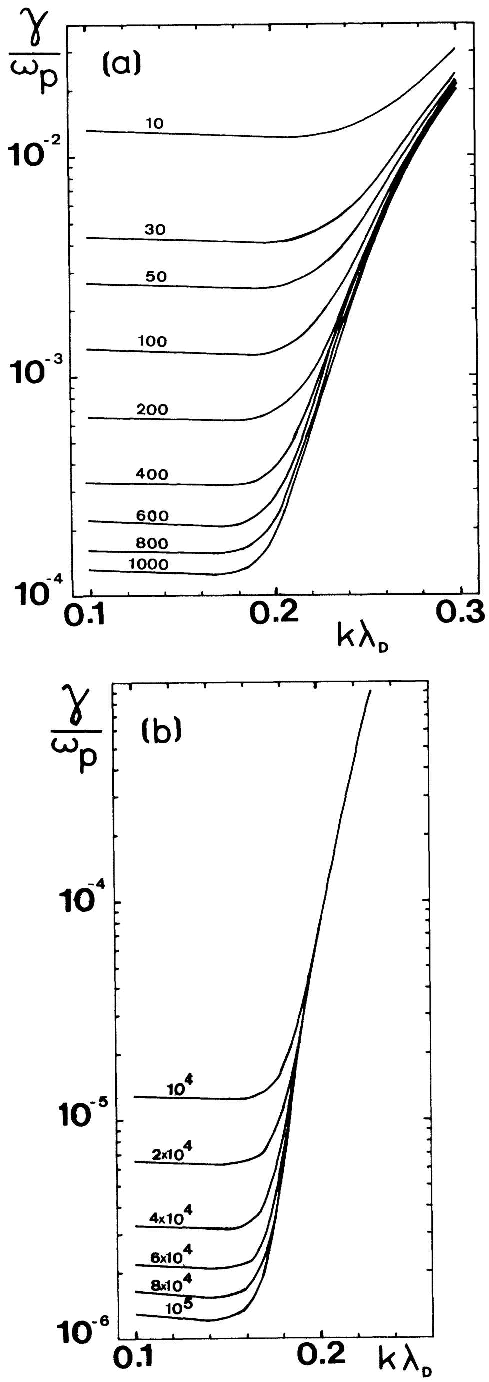

FIG. 2. Longitudinal dimensionless damping rates γ_L/ω_p vs $k\lambda_D$ for different values of the collisionality parameter λ_0/λ_D. (a) $\lambda_0/\lambda_D = 10, 30, 50, 100, 200, 400, 600, 800, 1000$; (b) $\lambda_0/\lambda_D = 10^4, 2\times10^4, 4\times10^4, 6\times10^4, 8\times10^4, 10^5$. The dashed line corresponds to the collisional contribution to the damping rate obtained from Eq. (22). As expected, the collisionless domain (in $k\lambda_D$ space) increases with λ_0/λ_D.

Our numerical results (Fig. 2) are very close to those of Ref. [10]. However, our results do not rely on the restrictive condition $g \ll 1$ of Ref. [10] [corresponding to $\lambda_D 4\pi/\lambda_0 \ln(\Lambda) Z \ll 1$ in our notation], which is required for the iterative method used there to converge. Therefore they remain valid for strongly collisional plasmas ($\lambda_D/\lambda_0 < 1$) which extend substantially the range of validity of the present model.

B. Transverse dispersion relation

In the high-frequency approximation the continuous fraction $F_0^{(1)}$ reads

$$F_0^{(1)} = \Omega_r^{-2}\left[2+\Gamma+2\left[\frac{q}{\Omega_r}\right]^2+\frac{3}{5}\Gamma\left[\frac{q}{\Omega_r}\right]^2 +2\left[\frac{q}{\Omega_r}\right]^4+\frac{3}{7}\Gamma\left[\frac{q}{\Omega_r}\right]^4\right] +i\Omega_r^{-1}\left[1+\frac{1}{5}\left[\frac{q}{\Omega_r}\right]^2+\frac{3}{35}\left[\frac{q}{\Omega_r}\right]^4\right] .$$

Isolating the real part of Eq. (20) we deduce the real part of the frequency:

$$\omega_r = \omega_p[1+(k\lambda_D S)^2]^{1/2} \times\{1+(k\lambda_D)^2[1+(k\lambda_D S)^2]^{-2} + (k\lambda_D)^4[1+(k\lambda_D S)^2]^{-3} \times[3-(1+(k\lambda_D S)^2]^{-1}+\cdots\}^{1/2} ,$$

where

$$S = \frac{l_s}{\lambda_D} \cong \frac{716}{\sqrt{T(eV)}}$$

($l_s = c/\omega_p$ is the collisionless skin depth). The imaginary part of Eq. (20) gives us the semicollisional transverse damping rate:

$$\gamma_T = \frac{-32\lambda_0\omega_p/\lambda_D}{3\sqrt{2\pi}+32\lambda_0/\lambda_D}\,\frac{\int_0^{y_{\max}} y^3 \mathrm{Re}(F_0^{(1,\nu)})\exp(-y)dy}{\int_0^{y_{\max}} y^3 \mathrm{Re}(F_0^{(1,\gamma)})\exp(-y)dy} , \tag{25}$$

where

$$F_l^{(1)} = \Gamma\,\mathrm{Re}(F_l^{(1,\gamma)}) + \mathrm{Re}(F_l^{(1,\nu)}) + i\,\mathrm{Im}(F_l^{(1)}) ,$$

with

$$\mathrm{Im}(F_l^{(1)}) = \left[4\sqrt{2}\left[\frac{\lambda_0\omega_r}{\lambda_D\omega_p}\right]y^{3/2} - 64(k\lambda_0)^2 y^4 \frac{(l+1)(l+3)}{4(l+2)^2-1}\mathrm{Im}(F_{l+1}^{(1)})\right]^{-1} ,$$

$$\mathrm{Re}(F_l^{(1,\gamma)})=[\mathrm{Im}(F_l^{(1)})]^2\left[4\sqrt{2}y^{3/2}\frac{\lambda_0}{\lambda_D}+\frac{64(l+1)(l+3)y^4}{4(l+2)^2-1}\times(k\lambda_0)^2\mathrm{Re}(F_{l+1}^{(1,\gamma)})\right],$$

and

$$\mathrm{Re}(F_l^{(1,\nu)})=[\mathrm{Im}(F_l^{(1)})]^2\left[(l+1)(l+2)+\frac{64(l+1)(l+3)y^4}{4(l+2)^2-1}\times(k\lambda_0)^2\mathrm{Re}(F_{l+1}^{(1,\nu)})\right].$$

The upper limit $y_{\max}$ of the integrals in Eq. (25) is taken as $(2y_{\max}v_t^2)^{1/2}<c$ in order to avoid the unphysical contribution of the Landau damping to high-frequency transverse modes. In the high-frequency approximation Eq. (25) reads

$$\gamma_T=-\frac{v_t a^2}{3\sqrt{2\pi}\lambda_0}[1+(ka\lambda_D)(2-2a^2)+(ka\lambda_D)^2(8-11a^2+6a^4)+\cdots],$$

where

$$a=[1+(k\lambda_D S)^2]^{-1/2}. \tag{26}$$

This result is original, as it incorporates for the first time corrections due to the thermal effects. We have checked numerically that for $k\lambda_D<0.3$, Eq. (25) gives the same result as Eq. (26) with a precision always better than 1%. For this reason the results do not need to be depicted on a graph for this low $k\lambda_D$ range while for the higher k's, more work is needed. In the low-frequency range the results, applied to Weibel instabilities, are widely discussed in Ref. [11]. Let us now compare our results with those obtained from the DO model. The DO model uses the Vlasov-Poisson set of equations with a discrete ionic distribution charge. The ion-electron collisions being exactly defined, this model may describe physical phenomena at any time scale. Keeping the leading terms (we neglect the thermal corrections) the ratio of the DO to VL [Eq. (26)] damping rates is

$$\frac{\gamma_{\mathrm{DO}}}{\gamma_L}=\frac{I}{\ln\Lambda}, \tag{27}$$

where

$$I=\frac{1}{\sqrt{\pi}}\int_{\eta_{\min}}^{\infty}\frac{\Delta}{\eta\{[(\omega/\omega_p)^2+2\eta^2+2\eta^3\Gamma]^2+4\eta^6\Delta^2\}}d\eta, \tag{28}$$

with $\eta=\omega/\sqrt{2}kv_t$ and $\Gamma(\eta)-i\Delta(\eta)$ being the Fried and Conte [18] function. The argument of the Coulomb logarithm corresponds to the ratio of the maximum to the minimum impact parameter, i.e., $\Lambda=p_{\max}/p_{\min}$. In the classical approximation (nondegenerate plasma electrons) $p_{\min}$ corresponds to the minimum distance approach, i.e., $p_{\min}=Ze^2/4\pi\epsilon_0 T$ and for degenerate electrons, $p_{\min}$, corresponds to the thermal de Broglie wavelength $\lambda_{\mathrm{the}}=h/(2\pi mT)^{1/2}$ and for which $\lambda_{\mathrm{the}}>Ze^2/4\pi\epsilon_0 T$. The maximum impact parameter corresponds to the Debye length, i.e., $p_{\max}=v_t/\omega_p$. In the high-frequency approximation, collisions at large distances have a slow time scale as compared to the rate of oscillation of electrons in the field. This tends to reduce the efficiency of the collision. Roughly [7], we can take into account these high-frequency effects by setting $p_{\max}=v_t/\omega_r$. The numerical comparison [Eq. (27)] gives identical results for low-frequency waves ($\omega_r\ll\omega_p$). For high-frequency waves ($\omega_r\geq\omega_p$) we obtain a qualitatively good agreement. Indeed, for $\omega_r=\omega_p$ the precision is better than 1%. When ω_r increases the discrepancy between the two results increases but remains relatively moderate in a large range of frequency values: For $\omega_r=10\omega_p$ and for an extremely large value $\omega_r=100\omega_p$ the precision is, respectively, about 3% and 10%.

VI. ANISOTROPIC EFFECTS

In this section we study the contribution of a weak velocity anisotropy induced by the spatial plasma inhomogeneity along the x axis to the high-frequency dielectric functions. For this, let us expand the unperturbed distribution function on the Legendre polynomials' basis up to the first order. That is sufficient to compute the leading anisotropic corrections:

$$f_s(y,v_x)=f_s^{(0)}(y)+\sqrt{3}(v_x/v)f_s^{(1)}(y).$$

After computing explicitly the source terms $S_l^{(0,\pm1)}$, the use of Eqs. (14)–(16) gives

$$\epsilon_L=1-\frac{256\pi\sqrt{2}v_t\lambda_0^2\omega_p^2}{3}\int_0^\infty y^{9/2}F_0^{(0)}F_1^{(0)}\frac{\partial f_s^{(0)}}{\partial y}dy+i\frac{16\pi\sqrt{2/3}v_t\omega_p^2\sin(\alpha)\lambda_0}{k}\int_0^\infty y^{1/2}F_0^{(0)}\left[5yf_s^{(1)}+2y^{5/2}\frac{\partial}{\partial y}\left[\frac{f_s^{(1)}}{y^{1/2}}\right]\right]dy, \tag{29}$$

$$\epsilon_T=1-i\frac{16\pi\omega_p^2v_t^2\lambda_0}{3\omega}\int_0^\infty\left[F_0^{(1)}\left[4y^3\frac{\partial f_s^{(0)}}{\partial y}-\frac{2\sqrt{6}kv_t\sin(\alpha)}{\omega}y^{5/2}f_s^{(1)}\right]+\frac{32\sqrt{3}\sin(\alpha)}{5}k\lambda_0 y^{11/2}F_0^{(1)}F_1^{(1)}\frac{\partial}{\partial y}\left[\frac{f_s^{(1)}}{y^{1/2}}\right]\right]dy, \tag{30}$$

where

$$\epsilon_T = \epsilon_{T_1} = \epsilon_{T_2} \ .$$

From Eqs. (19) and (20) we can deduce straightforwardly the dispersion relations which contain both thermal and anisotropic corrections. Assuming the plasma very close to the local thermodynamic equilibrium ($f_0^{(0)} = f_M$ and $f_s^{(1)}/f_M \sim \epsilon$), we obtain, after some algebra, a negligible anisotropic correction $(kv_t/\omega_r)^3 M_1^2$ for the real frequency $\omega_r(k)$. We have used the following notation: $M_i^n = \int_0^\infty f_s^{(i)} y^n dy$. For the damping rates the anisotropic contributions will appear under the form $\omega_r(\epsilon M_1^n)/kv_t$ and $(\epsilon^2 M_2^p) \sim (\epsilon^2 M_1^{p+2})$. In Ref. [11], where the quasistatic approximation $(\omega_r/kv_t) \sim \epsilon$ is used, both contributions of the anisotropies are of the same order of magnitude, of order ϵ^2. It should be noted, however, in the expression of the instability rate, the moment of $f_s^{(2)}$ is of an order much greater than that of $f_s^{(1)}(p+2 \gg n)$, hence, only the contribution of $f_s^{(2)}$ strongly dominates, while in the present calculation, only the first term, of order ϵ, in $f_s^{(1)}$ is kept and the damping rates are found as

$$\gamma_L = -\frac{v_t}{3\sqrt{2\pi}\lambda_0} \left[1 - 2 \left(\frac{kv_t}{\omega_p} \right)^2 + \left(\frac{4\pi^3}{3} \right)^{1/2} k v_t^4 \omega_p^{-1} \sin(\alpha) \times M_1^{-1/2} + \cdots \right] , \quad (31)$$

$$\gamma_T = -\frac{v_t a^2}{3\sqrt{2\pi}\lambda_0} [1 + (2-2a^2)(ka\lambda_D)^2 + (12\pi^3)^{1/2} ka\lambda_D v_t^3 \sin(\alpha) \times M_1^{-1/2} + \cdots] . \quad (32)$$

We note that, when the wave vector **k** is perpendicular to the inhomogeneity direction ($\alpha = 0$), the anisotropic corrections vanish as expected. Let us compare explicitly the thermal corrections to the anisotropic ones. For this, we use the Chapman-Enskog anisotropic function [19]:

$$f_s^{(1)}(y) = \frac{4\lambda_0}{\sqrt{3}L_T} y^2 (4-y) \exp(-y) \ .$$

We recall that this solution is valid for mean free paths λ_0 much smaller than the characteristic temperature scale length $L_T = T|dT/dx|^{-1}$, i.e., $\lambda_0/L_T \sim \epsilon$. We deduce thus, respectively, the anisotropic terms in Eqs. (31) and (32):

$$\frac{3\sqrt{2\pi}}{4} \sin(\alpha)(k\lambda_D \lambda_0/L_T) \ ,$$

$$\frac{9\sqrt{2\pi}}{4} \sin(\alpha)(ka\lambda_D \lambda_0/L_T) \ .$$

Using the high-frequency approximation $[k\lambda_D = (kv_t/\omega_p) \sim \epsilon$ and $ka\lambda_D = (kv_t/\omega_r) \sim \epsilon]$ and the Chapman-Enskog approximation $[(\lambda_0/L_T) \sim \epsilon]$, we observe that both the thermal and the anisotropic corrections are of the same order, ϵ^2. Therefore, for weak anisotropic plasmas we have to take into account both corrections. For strongly inhomogeneous plasmas $[(\lambda_0/L_T) \sim 0.1]$, the nonlocal effects [20] tend to reduce the distribution function moments M_i^n. Hence the correction terms decrease whereas the damping rate $[\gamma_{L,T} \sim (M_0^{1/2})^{-1}]$ slightly increases. We expect that in the plasmas with strong anisotropy the corrections due to anisotropic effects should be much higher than those of the thermal effects.

VII. SUMMARY

In this paper we have derived from the Vlasov-Landau equation the transverse and the longitudinal electric function, valid for the whole collisionality range. We have deduced the damping rates [Eqs. (24) and (25)] for isotropic plasmas. For Maxwellian plasmas, in the asymptotic high-frequency and collisional approximations, an explicit form for these damping rates is computed with thermal corrections up to the fourth order. In the low-frequency limit, the Spitzer-Härm [17] conductivity is recovered and for the transverse damping rates, a good agreement is obtained with the high-frequency Dawson-Oberman model [7]. We have also numerically computed longitudinal damping rates versus the wave number k for different values of the collisionality parameter λ_0/λ_D. We have pointed out the continuous transition between the collisional and the Landau collisionless limit. The effects of the anisotropy of velocity space on the dielectric functions are studied. It is shown that those effects contribute to the same order as the thermal effects for the collisional damping rates. An extension of the present work is in order. A quantitative analysis of the high $k\lambda_D$ values would be of some interest in the whole collisional range for both high-frequency transverse and longitudinal waves as well as low-frequency ion acoustic waves [21]. Moreover, the strong anisotropy effect on the plasma modes is also of interest just like the ion acoustic instability in the semicollisional regime.

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